

Umklapp-Assisted Electron Transport Oscillations in Metal Superlattices

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We consider a superlattice of parallel metal tunnel junctions with a spatially non-homogeneous probability for electrons to tunnel. In such structures tunneling can be accompanied by electron scattering that conserves energy but not momentum. In the special case of a tunneling probability that varies periodically with period a in the longitudinal direction, i.e., perpendicular to the junctions, electron tunneling is accompanied by “umklapp” scattering, where the longitudinal momentum changes by a multiple of h/a . We predict that as a result a sequence of metal-insulator transitions can be induced by an external electric- or magnetic field as the field strength is increased.

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To design novel electrical conductors in the form of artificially structured materials remains one of the most important tasks of nanoscience. This is because progress in this type of “quantum engineering” may lead to new and better electronic devices. Multilayered systems are a widely used material of this type, with semiconductor superlattices arguably the most prominent example.¹ Work on semiconductor superlattices with spatially modulated properties on the sub-micron scale started already 40 years ago, following the pioneering work of Esaki and Tsu.² The early focus on semiconductors was natural, since the de-Broglie wavelength of their conduction electrons is typically large enough to be comparable to the period of then feasible superlattices. Qualitatively new effects based on quantum interference phenomena — still mostly absent in metal superlattices³ — could therefore be predicted and observed.¹

More recent developments have lead to engineered conductors such as quantum dots, nano-wires, and other “nanosolids”, which could be useful components in novel superlattice architectures.⁴ Here, we focus on nano-wires and note that many are good metals with a high conductivity due to ballistic electron transport. However, the de Broglie wavelength of their electrons is typically much too small compared to the modulation period a of a nano-wire based superstructure for quantum interference effects to occur. On the other hand, we will show in this Letter that a prominent interference effect of a different origin emerges in such structures under the realistic assumption that $k_F a \gg 1$, so that the quasi-classical approximation is valid and hence the electron energy dispersion can be linearized.

The two-dimensional (2D) superlattice structure to be considered is sketched in Fig. 1. It comprises a set of 1D wires coupled by electron tunneling in such a way that the probability for tunneling varies periodically along the direction of the wires. For this structure we will show

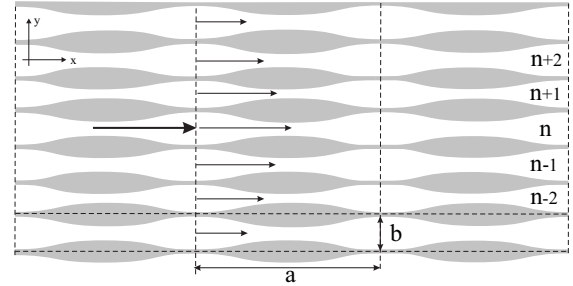


FIG. 1: Sketch of the considered 2D superlattice, comprising an array (period b) of parallel nanowires coupled by inter-wire electron tunneling with a periodically modulated tunneling strength (period a).

that when a magnetic field is applied perpendicular to the 2D plane — or when an electric field is applied in the plane and perpendicular to the wires — a series of metal-insulator transitions occur with respect to the inter-wire hopping transport of electrons as the strength of the external field increases.

In order to understand how an increasing electric or magnetic field can induce a series of metal-insulator transitions, it is instructive to first consider a single, isolated wire, where electrons move freely along the longitudinal direction and occupy a 1D band of “longitudinal” energies. In the transverse direction, they are confined to the wire and occupy a single, discrete “transverse” energy level. Now, disregard for a moment the longitudinal motion and focus on the transverse electron dynamics in a set of identical wires, aligned in parallel and each with the same transverse level occupied. By switching on a longitudinally uniform probability for electrons to tunnel to adjacent wires, these previously degenerate energy levels will form a band, which allows transverse motion between wires. Hence, in the absence of an external field the su-

perlattice is effectively a two-dimensional (non-isotropic) metal.

If we now apply an external electric field $\vec{\mathcal{E}}$ perpendicular to the wires, the transverse energy levels will be shifted out of resonance so that band motion in the transverse direction is prevented by Wannier-Stark localization of the electron states.⁵ However, if the tunneling probability can be made to vary periodically along the wires, the situation is qualitatively modified. This is because (i) the longitudinal and transverse motion of the electrons can no longer be separated, and (ii) the longitudinal momentum only has to be conserved modulo $\hbar G$, where $G = 2\pi/a$ and a is the modulation period. Although the total energy is still conserved when electrons tunnel between wires, energy can now be shifted from the longitudinal to the transverse motion in “umklapp processes” that involve discrete changes of longitudinal momentum. It follows that the transverse energy-level shifts can be compensated and band motion restored for a discrete set of electric-field values.

The effect of umklapp-assisted resonant tunneling is controlled by the dimensionless parameter $\phi = \Phi/\Phi_0$, where $\Phi = H_{\text{eff}} ab$ is the flux of an “effective” magnetic field $H_{\text{eff}} = (c/v_F)|\vec{\mathcal{E}}|$ through a superlattice “unit cell” of area ab (see Fig. 1) and $\Phi_0 = hc/e$ is the magnetic flux quantum. The result is the same if instead a magnetic field H is applied, except that now $\Phi = Hab$. Resonant tunneling occurs when $\phi = p/q$ is a rational number (p and q are integers) but, as we will discuss below, band motion is only possible when ϕ is an integer (i.e., for $q = 1$).

In Fig. 2 we illustrate the effect of umklapp-assisted resonant tunneling when $\phi = 1$ (panel a) and $\phi = 1/2$ (panel b). While in case (a) the resonance condition is fulfilled for neighboring wires, in case (b) every second wire is in resonance and resonant tunneling occurs via virtual states on the intermediate wire. In this case contributions to the total tunneling amplitude from a number of different paths through various virtual states have to be summed up. The contributions from the two paths identified by arrows in Fig. 2(b) have equal magnitude but different signs (since the two virtual levels have mirror symmetry with respect to the resonant levels). By generalizing this argument one finds that tunneling between the resonant levels becomes completely suppressed by destructive interference in this case (if direct hopping between next-nearest neighbors is neglected). In the general case of a rational flux, $\phi = p/q$, the transverse energies are resonant for wires separated by a distance $\Delta y = qb$, where b is the superlattice period. Although it would be quite difficult to explicitly consider the destructive interference between all the possible paths for large q , we are nevertheless able to prove below that resonant hopping is completely suppressed for any (non-integer) rational value of the parameter ϕ .

We consider an infinite superlattice structure (see Fig. 1) subject to a constant electric field $\vec{\mathcal{E}} = -\mathcal{E}\hat{y}$. In the quasiclassical limit ($k_F a \gg 1$) large momentum

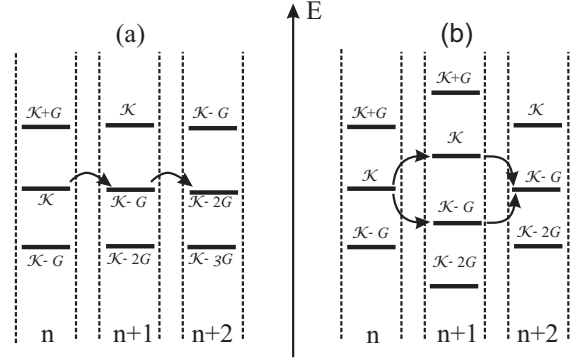


FIG. 2: Transverse energy levels involved in resonant inter-wire tunneling (thick bars) in three neighboring wires (n , $n+1$, $n+2$). The total electron energy is the sum of a longitudinal part $\propto \mathcal{K}$ (linear spectrum assumed) and a transverse part, which (for fixed \mathcal{K}) is shifted from one wire to the next by an amount proportional to the dimensionless flux ϕ of an external electric or magnetic field. Since \mathcal{K} in the periodically modulated superlattice system of Fig. 1 is only conserved modulo G , energy can be shifted between the longitudinal and transverse parts in a tunneling event that conserves the total energy. This is why a sequence of transverse levels is shown for each wire. For integer values of the flux parameter, as in (a) where $\phi = 1$, resonant transmission (arrows) between states on adjacent wires are energetically allowed. In (b), where $\phi = 1/2$, resonant transitions occur between states on every second wire. The two paths through virtual states on the intermediate wire shown (arrows) contribute with equal amplitude but opposite signs to the total transition amplitude, which when all paths are considered turns out to be zero due to destructive interference.

transfers ($\Delta p \sim p_F$) may be neglected, which justifies a linearization of the energy dispersion related to the longitudinal motion (in each wire). Also, the Hamiltonian may be split into separate parts for left- and right-moving electrons, which can be treated independently. Hence, the spectrum E (measured from the Fermi level) and the stationary wave functions for the problem at hand can be found by solving the Schrödinger equation

$$-E_0 \left(i \frac{\partial}{\partial x} + 2\pi n \phi \right) \varphi_n(x) + v(x) [\varphi_{n+1}(x) + \varphi_{n-1}(x)] = E \varphi_n(x). \quad (1)$$

Here x is the dimensionless coordinate along the wire(s), $v(x) = v(x+1)$ is the periodic potential responsible for electron transitions between the wires, and $E_0 = \hbar v_F / a$. Equation (1) is unchanged if one instead considers a superlattice subject to a constant and not too strong (see below) magnetic field $\vec{H} = H\hat{z}$. In both cases ϕ is the dimensionless flux of the relevant external field — the magnetic field H or the effective magnetic field $H_{\text{eff}} = (c/v_F)\mathcal{E}$ — through a unit cell of the superlattice structure.

We will now proceed by solving Eq. (1) exactly for an arbitrary potential $v(x)$. The first step is to note that

according to Bloch's theorem $\varphi_n(x) = \exp(i\mathcal{K}x)u_n(x)$, where \mathcal{K} is the (dimensionless) quasimomentum, $-\pi < \mathcal{K} < \pi$, and $u_n(x) = u_n(x+1)$ is a periodic function. It is convenient to define the auxiliary function $u(n) \equiv u_n(x = -1/2)$. This function obeys the equation,

$$u(p) = e^{i(\varepsilon - \mathcal{K} + 2\pi\phi p)} \sum_n e^{-i\theta(p-n)} J_{p-n}(A) u(n), \quad (2)$$

where $J_n(x)$ is a Bessel function, $\varepsilon = E/E_0$, while A and θ are defined through the relations

$$\frac{2}{E_0} \int_{-1/2}^{1/2} dy v(y) e^{2\pi i \phi y} \equiv A e^{i\xi}, \quad \theta = \pi/2 + \pi\phi + \xi. \quad (3)$$

Equation (2) determines the energy spectrum and the wave functions in our problem. The structure of the spectrum strongly depends on the nature of the number ϕ . If ϕ has a *non-integer* value, the eigenenergies and eigenstates are labeled by the three quantum numbers \mathcal{K} , m , and r ; the quasi-momentum \mathcal{K} and the (integer) band index m refer to the longitudinal motion along the x -axis while the integer r is related to the transverse motion in the y -direction. The dispersion law reads

$$E_{m,r}(\mathcal{K}) = E_0 (\mathcal{K} + 2\pi m - 2\pi\phi r), \quad m, r = 0, \pm 1, \dots \quad (4)$$

The energy-level distribution depends crucially on whether or not the noninteger ϕ is a rational number. If it is a rational number, $\phi = p/q$, one notes that $E_{m,r}(\mathcal{K}) = E_{m+Mp, r+Mq}(\mathcal{K})$ for a given quasi-momentum \mathcal{K} and any integer M . This results (for each \mathcal{K}) in a set of infinitely degenerate, equidistant energy levels. If ϕ is an irrational number, on the other hand, the energy levels are homogeneously distributed forming a discrete spectrum that is said to be everywhere dense.

The eigenfunctions can be found from Eq. (2) rewritten as

$$u_{m,r}(n) = e^{-i\xi(n-r)} J_{n-r}(\gamma), \quad \gamma = \frac{A}{2 \sin \pi \phi}, \quad (5)$$

and have the same form for both rational and irrational values of ϕ . Therefore, for any non-integer ϕ , all states are localized near a particular wire, r , within a localization radius R_{loc} defined as

$$\left(\frac{R_{\text{loc}}}{b} \right)^2 \equiv \sum_n n^2 J_{n-r}^2(\gamma) - \left[\sum_n n J_{n-r}^2(\gamma) \right]^2 = \frac{\gamma^2}{2}. \quad (6)$$

It is remarkable that this result holds even when the flux parameter ϕ is a (non-integer) rational number, since in this case our superlattice structure has translational symmetry in the y -direction. Accordingly, bands of electron states with infinite extension in this direction should form (see, e.g., Ref. 6). However, even though the transverse energy levels of the parallel wires periodically are in resonance, this does not happen. The reason is a fully destructive interference between the probability amplitudes

for resonant hopping along different paths, as illustrated for the special case of $\phi = 1/2$ in Fig. 2(b).⁷

In case ϕ is an *integer*, not only does the external field become effectively periodic in the y -direction but a band of extended states also form [$R_{\text{loc}} \rightarrow \infty$, according to Eqs. (5) and (6)]. Therefore, in addition to the two ‘‘longitudinal’’ quantum numbers \mathcal{K}, m a continuous quasimomentum $\tilde{\mathcal{K}}$, where $-\pi < \tilde{\mathcal{K}} < \pi$, must be used to label the transverse motion along the y -axis. The dispersion law, which is found from Eq. (2), reads

$$E_m(\mathcal{K}, \tilde{\mathcal{K}}) = E_0 \left(\mathcal{K} + 2\pi m + A \sin(\tilde{\mathcal{K}} + \theta) \right). \quad (7)$$

Thus, for each pair (\mathcal{K}, m) of eigenvalues related to the longitudinal motion, the transverse energies spread into a band of width $\delta E = 2AE_0$. The corresponding eigenstates are of the plane wave type, $u_{\tilde{\mathcal{K}},m}(n) = \exp(-i\tilde{\mathcal{K}}n)$ and, as a consequence, are delocalized in both the longitudinal and transverse directions.

The detailed features of the energy spectrum influence various physical quantities in essential ways. Here, we consider the linear response of the system to a weak ac external electrical field $\vec{\mathcal{E}}(t) = \mathcal{E}_0 \cos \omega t \hat{y}$. The interaction with the field adds a perturbation term $\mathcal{H}_{\text{int}}(t) = \mathcal{H}^{(\text{int})} \cos \omega t$ to the Hamiltonian, where

$$\mathcal{H}^{(\text{int})} = eb\mathcal{E}_0 \sum_n n \int dx \Phi_n^\dagger(x) \Phi_n(x), \quad (8)$$

and $\Phi_n^\dagger(x)$ [$\Phi_n(x)$] is a field operator that creates [destroys] an electron at point x in the n -th wire and obeys the standard anti-commutation relations.

Absorption of the ac electric field is proportional to the real part of the conductivity $\sigma(\omega)$, which in linear response theory has the form

$$\sigma(\omega) = \frac{1}{\mathcal{E}_0 L} \sum_{\alpha, \beta} I_{\alpha, \beta} \mathcal{H}_{\beta, \alpha}^{(\text{int})} \frac{f(E_\alpha) - f(E_\beta)}{E_\alpha - E_\beta - \hbar(\omega - i\nu)}. \quad (9)$$

Here ν is a phenomenological relaxation rate, L is the sample length in the longitudinal direction, $f(E)$ is the Fermi-Dirac distribution function, and $I_{\alpha, \beta}$ is the matrix element of the current operator

$$\hat{I} = \frac{ie}{\hbar N} \sum_n \int dx v(x) \left[\Phi_{n+1}^\dagger(x) \Phi_n(x) - H.c. \right], \quad (10)$$

where N is a total number of wires in superlattice.

Standard calculations lead to an electrical conductivity of the form

$$\sigma(\omega) = \frac{ie^2}{\hbar} \frac{ab}{(\hbar v_F)^2} \sum_n |v_n|^2 \times \left[\frac{1}{2\pi(n - \phi) - t_0(\omega - i\nu)} - \frac{1}{2\pi(n - \phi) + t_0(\omega - i\nu)} \right], \quad (11)$$

where $t_0 = a/v_F$ and v_n is a Fourier component of the potential $v(x)$. In the low frequency limit, $\omega \ll \nu$, the

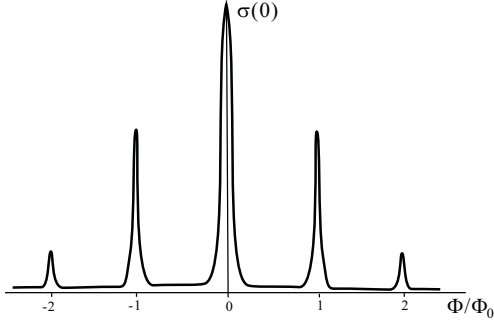


FIG. 3: Schematic behavior of the low-frequency transverse conductivity, $\sigma(0)$, as a function of magnetic field flux Φ through a unit cell of the superlattice structure shown in Fig. 1; $\Phi_0 = hc/e$ is the flux quantum.

electrical conductivity can be approximated as

$$\sigma(0) = G_0 \frac{ab}{(\hbar v_F)^2} \sum_n |v_n|^2 \frac{t_0 \nu}{[2\pi(n - \phi)]^2 + (t_0 \nu)^2}, \quad (12)$$

where $G_0 = 2e^2/h$ is the conductance quantum. It follows that in this limit the conductivity plotted as a function of the number of flux quanta per unit cell in the superlattice, ϕ , has a set of peaks corresponding to integer numbers, $\phi = 0, \pm 1, \pm 2, \dots$ (see Fig. 3). The scale of the fluctuations are determined by the ratio between successive maxima $[\sigma_l(0), \phi = l]$ and minima $[\sigma_{l+1/2}(0), \phi = l + 1/2]$ of the conductivity,

$$\frac{\sigma_l(0)}{\sigma_{l+1/2}(0)} \sim \frac{1}{(t_0 \nu)^2}; \quad \sigma_{l+1/2}(0) \sim G_0 t_0 \nu (R_{\text{loc}}/b)^2 \quad (13)$$

where R_{loc} is the localization radius, Eq. (6). Hence, if $t_0 \nu \ll 1$, the field dependence of the absorption as well as the conductivity has a pronounced peak structure as schematically shown for the conductivity in Fig. 3.

In our analysis we have used the Schrödinger equation (1), which has a linear energy spectrum. For this approximation to be valid the longitudinal momentum fluctuations associated with umklapp-assisted resonant transmission of electrons between adjacent wires must be small on the scale of the Fermi momentum p_F . The total momentum change due to such processes is restricted by the relaxation time $\tau_r \sim 1/\nu$, which corresponds to coherent tunneling through $N_\nu \sim v_t \tau_r / b$ wires, where $v_t \sim AbE_0/\hbar$ is the electron velocity in the transverse direction. Reasonable estimates for the “tunneling” parameter, $A \sim 10^{-3}$, and for the superlattice periods, $a, b \sim 10 \mu\text{m}$, give an upper limit of $\tau_r \sim 10 \text{ ns}$ for a relaxation time consistent with an approximately linear spectrum. On the other hand, the criterion that the

electron motion is ballistic gives a lower limit for the relaxation time of $\tau_r \sim b/v_t \sim 0.1 \text{ ns}$.

An additional requirement for the Schrödinger equation (1) to be valid must be fulfilled if an external magnetic field is applied to the system. This is because the cyclotron motion of the electrons can be neglected (and a term quadratic in H dropped from Eq. (1)) only if

$$H \leq \frac{E_F}{E_0} \frac{\Phi_0}{2\pi ab N_\nu}. \quad (14)$$

For a relaxation time $\tau_r \sim 1 \text{ ns}$ this restriction corresponds to $H \leq 6 \text{ T}$.

The single-particle approach used in our analysis neglects the Coulomb interaction between electrons. In principle, however, a Coulomb blockade of inter-wire tunneling might prevent the formation of extended electron states in the transverse direction.⁸ To avoid such a blockade the electrostatic charging energy of the wire, $E_{el} \sim e^2/L$, should be smaller than than width, $\delta E \sim AE_0$, of the transverse energy band. Using our estimates for the relevant parameters, this corresponds to a lower limit of order $100 \mu\text{m}$ for the length L of the nano-wires.

Finally we estimate the required strengths of the external electric- and magnetic fields by noting that a dimensionless flux of $\phi = 1$ corresponds to $\mathcal{E} \sim 0.05 \text{ V/cm}$ or $H \sim 0.4 \text{ T}$. Such field strengths can easily be applied in an experiment.

In conclusion, we have shown that the transport properties of a superlattice comprising a set of parallel metallic nano-wires coupled by tunneling in such a way that the tunneling probability varies periodically along the wires drastically differ from the predictions of linear transport theory. In particular, an electric-field induced sequence of metal-insulator transitions gives rise to a highly non-linear current-voltage characteristics, while the sensitivity to a magnetic field leads to large-magnitude oscillations of the magneto-conductance. Importantly, these phenomena are manifest in comparatively weak external fields. Hence, in relatively weak fields interference phenomena give rise to pronounced mesoscopic features in the transport properties of the studied metallic superlattices. Such superlattice structures could, e.g., be realized using arrays of nanowires similar to those that have been proposed for memory⁹ and mechanical single-electron transistor¹⁰ applications.

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